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presents:

On Solution Discovery via Reconfiguration

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The framework

Let Π be a *vertex subset problem*, i.e., a problem defined on undirected graphs such that a solution consists of a subset of vertices.

Π-DISCOVERY

Input:
- graph G
- configuration $S \subseteq V(G)$; $k = |S|$ – *each vertex in S contains a token*, multiple tokens occupying the same vertex are forbidden, *S is not necessarily a solution for* Π
- budget $b \in \mathbb{N}$

Question: Decide if we can move the tokens on S using at most b moves such that the resulting set is a solution for Π.

COLORING DISCOVERY

Input:
- graph G
- (non-proper) k-coloring $\phi : V \to [k]$
- budget $b \in \mathbb{N}$

Question: Decide whether there is a transformation of the given coloring into a proper k-coloring by using at most b recoloring steps.

Underlying problems of interest

VERTEX COVER

Input: Graph G; $k \in \mathbb{N}$

Question: Decide whether G admits a set C of at most k vertices such that for every edge $\{u, v\} \in E(G)$ we have $\{u, v\} \cap C \neq \emptyset$.

INDEPENDENT SET

Input: Graph G; $k \in \mathbb{N}$

Question: Decide whether G contains a set of pairwise non-adjacent vertices of size k.

DOMINATING SET

Input: Graph G; $k \in \mathbb{N}$

Question: Decide whether G admits a set D of at most k vertices such that every vertex in $V(G)$ is either in D or has a neighbor in D.

COLORING

Input: Graph G; $k \in \mathbb{N}$

Question: Decide whether G admits a proper k-coloring, i.e., a mapping $\phi : V \to [k]$ such that whenever $\{u, v\} \in E(G)$ then $\phi(u) \neq \phi(v)$.

Reconfiguration steps / Token moves

For token configurations S, $T \subseteq V(G)$ we have the following models of token moves:

token jumping	A token jumps from $u \in V(G)$ to $v \in V(G)$ if $u \in S$, $v \notin S$, $v \in T$, and $u \notin T$
token sliding	Token jumping with $\{u, v\} \in E(G)$
token addition	A token is added to vertex $v \in V(G)$ if $v \notin S$ and $v \in T$

For COLORING DISCOVERY:

color swapping	(u, v)-color-swap allows for the swap of colors between two arbitrary vertices $u \in V(G), v \in V(G)$, where $u \neq v$
color sliding	Color swapping with $\{u, v\} \in E(G)$
color flipping	In each step, we can change the color of any vertex to any color in the color set $[k]$

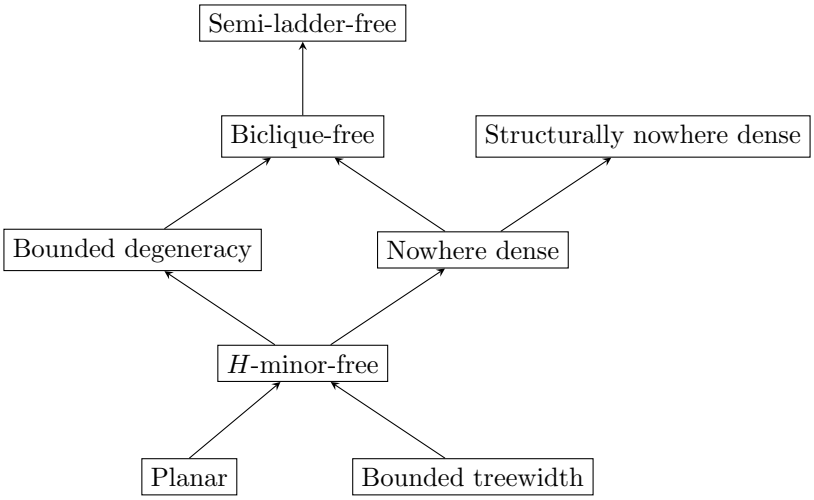


Figure 1: The considered graph classes. Arrows indicate inclusion.

Results

Vertex Cover Discovery (VCD)

- The problem is NP-complete even on planar graphs of maximum degree four and W[1]-hard for parameter b, even on 2-degenerate bipartite graphs.
- The problem is XP for parameter treewidth (that is, polynomial-time solvable on every fixed class of bounded treewidth), FPT for parameter k on general graphs and FPT for parameter b when restricted to structurally nowhere dense classes of graphs.

Independent Set Discovery (ISD)

- The problem is NP-complete even on planar graphs of maximum degree four, W[1]-hard for parameter $k + b$ even on graphs excluding $\{C_4, \ldots, C_p\}$ as induced subgraphs (for any constant p), and W[1]-hard for parameter b even on 2-degenerate bipartite graphs.
- The problem is XP for parameter treewidth, FPT for parameter k on d-degenerate and nowhere dense classes of graphs as well as FPT for parameter b on structurally nowhere dense classes of graphs.

Dominating Set Discovery (DSD)

- The problem is NP-complete even on planar graphs of maximum degree five, W[1]-hard for parameter $k + b$ even on bipartite graphs, and W[1]-hard for parameter b even on 2-degenerate graphs.
- The problem is XP for parameter treewidth, FPT for parameter k on biclique-free and semi-ladder-free graphs as well as FPT for parameter b on structurally nowhere dense classes of graphs.

Coloring Discovery (CD)

- The problem is NP-complete for every $k \geq 3$ even on planar bipartite graphs (that is, para-NP-hard for parameter k), W[1]-hard for parameter treewidth, and W[1]-hard for parameter $k + b$ on general graphs.
- The problem is polynomial-time solvable for $k = 2$ and FPT parameterized by $k+b$ on structurally nowhere dense classes of graphs.